the integrand readily integrable and results in an expression that is simple in form and computationally efficient.

Acknowledgments

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Starting Point of Curvature for Reflected Diffracted Shock Wave

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Introduction

THE diffraction of a normal shock wave past a small bend was considered by Lighthill.[1] The analogous problem of a shock hitting the wall obliquely together with the reflected shock has been considered by Srivastava and Chopra.[2] Srivastava and Chopra[2] determined the pressure distribution on the wall. The present work is concerned about the start of curvature of the reflected diffracted shock wave for the case when the relative outflow behind the reflected shock before diffraction is supersonic. The corresponding work for the normal shock has been carried out by Skews.[3]

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Mathematical Formulation

In Fig. 1 the velocity, pressure, density, and sound velocity are denoted by subscripts 0 ahead of incident shock, 1 in the intermediate region, and 2 behind the reflected shock. U in the figure denotes the velocity of the point of intersection of the incident and reflected shock and δ is the angle of bend. The relations across incident and reflected shock for $\gamma = 1.4$, where γ is the ratio of specific heats, are as follows.

Across the incident shock:

$$\begin{aligned} q_1 &= \frac{5}{6} U \sin\alpha_0 \left(1 - \frac{a_0^2}{U^2 \sin^2\alpha_0}\right) \\ p_1 &= \frac{5}{6}\rho_0 \left(U^2 \sin^2\alpha_0 - \frac{a_0^2}{7}\right) \\ \rho_1 &= \frac{6\rho_0}{1 + \left(5a_0^2/U^2 \sin^2\alpha_0\right)}, \qquad a_0 = \sqrt{\gamma p_0/\rho_0} \end{aligned} \tag{1a}$$

Across the reflected shock:

$$\begin{aligned} \bar{q}_2 &= \bar{q}_1 + \frac{5}{6}\left(U^* - \bar{q}_1\right)\left\{1 - \frac{a_1^2}{\left(U^* - \bar{q}_1\right)^2}\right\} \\ p_2 &= \frac{5}{6}\rho_1\left\{\left(U^* - \bar{q}_1\right)^2 - \left(a_1^2/7\right)\right\} \\ \rho_2 &= \frac{6\rho_1}{\left\{1 + 5a_1^2/\left(U^* - \bar{q}_1\right)^2\right\}} \\ \bar{q}_2 &= q_2 \sin\alpha_2, \qquad U^* = U \sin\alpha_2 \\ \bar{q}_1 &= -q_1 \cos(\alpha_0 + \alpha_2), \qquad a_1 = \sqrt{(\gamma p_1/\rho_1)} \end{aligned} \tag{1b}$$

Srivastava and Ballabh[4] have proved that the intermediate region (region between the incident and reflected shock) would remain undisturbed for all incident shock strengths after the shock configuration has crossed the corner. This result has received experimental confirmation.[5] Let the velocity, pressure, density and entropy at any point behind the reflected diffracted shock be $\boldsymbol{q}_2'$, p_2', ρ_2', and S_2'. Choose X and Y axes with the origin at the corner and the X axis along the original wall produced. Then the equations for conservation of mass and momentum can be written as

$$\frac{\mathrm{D}\rho_2'}{\mathrm{D}t} + \rho_2' \operatorname{div} \boldsymbol{q}_2' = 0 \tag{2}$$

$$\frac{\mathrm{D}\boldsymbol{q}_2'}{\mathrm{D}t} + \frac{1}{\rho_2'}\nabla p_2' = 0 \tag{3}$$

If there is no heat conduction or radiation, the entropy, satisfies the equation $\mathrm{D}S_2'/\mathrm{D}t = 0$. Using Lighthill's linearized theory[1] and the transformations

$$\begin{aligned} x &= \frac{X - q_2 t}{a_2 t}, \qquad y = \frac{Y}{a_2 t} \\ p &= \frac{p_2' - p_2}{a_2 \rho_2 q_2}, \qquad \frac{\boldsymbol{q}_2'}{q_2} = (1 + u, v) \end{aligned} \tag{4}$$

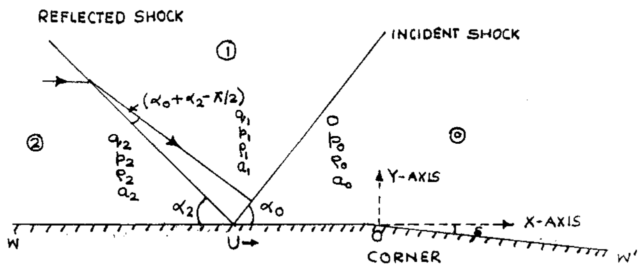

Fig. 1 Schematic drawing for oblique shock configuration passing over a small bend.

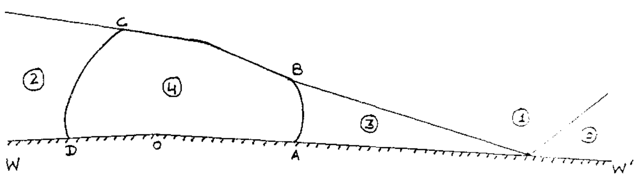

Fig. 2 Shape of disturbed region after diffraction, $(U - q_2 / a_2) > 1$, $q_2 / a_2 < 1$.

Eqs. (2) and (3) and the entropy equation yield a single second-order differential equation in p, viz.,

$$\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right) = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1\right)\left(x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y}\right) \qquad (5)$$

In (x, y) coordinates, the origin is at a point on the original wall produced, the corner is at $[-(q_2/a_2), 0]$. The coordinates of the point of intersection of the incident and reflected shock is $[(U - q_2)/a_2, 0]$. The straight portion of the reflected shock lies along a fixed line $x = k - y \cot \alpha_2$ where $k = (U - q_2)/a_2$.

The characteristics of Eq. (5) are tangents to the unit circle $x^2 + y^2 = 1$ and this ensures the disturbed region after diffraction, which is region 4 in Fig. 2. The pressure distribution on the wall has been determined by Srivastava and Chopra.[2]

Region 3 in Fig. 2 is the region of constant flow and conditions here are known from the changed angle of incidence, equal to $\alpha_0 + \delta$. The parameters chosen for the calculations are such that $(U - q_2)/a_2 > 1$ and $q_2/a_2 < 1$ and the configuration in such a case will be as in Fig. 2. Referring to Fig. 2 the coordinates of the point of intersection of C and B with the unit circle is given by

$$\begin{array}{ll} (K \sin \alpha_2 - K' \cos \alpha_2), & (K \cos \alpha_2 + K' \sin \alpha_2) \\ (K \sin \alpha_2 + K' \cos \alpha_2), & (K \cos \alpha_2 - K' \sin \alpha_2) \end{array} \qquad (6)$$

where

$$K = \left(\frac{U - q_2}{a_2}\right) \sin \alpha_2 = k \sin \alpha_2, \qquad K' = \sqrt{1 - K^2}$$

C and B are the points for the start of curvature in the reflected diffracted shock.

Discussion of Results

For given shock strength and angle of incidence, the angle of reflection has been determined by using Bleakney and Taub's equations.[6] The other parameters involved in determining Eq. (6) have been obtained from Eq. (1).

The coordinates of the intersecting points for different incident shock strengths ($0 \le \xi \le 1, \xi = p_0/p_1$) and angles of incidence α_0 equal to 10, 20, and 30 deg have been obtained. The angles formed by the wall and line joining the corner and the point of intersections have been determined. For C let it be m_1 and for B let it be m_2.

In Fig. 3, m_1 is plotted against ξ. The curve shows a minimum at $\xi = 0$, and then it starts increasing, reaches a maximum, and starts decreasing for lower incident shock strengths (higher ξ). This trend is true for all three cases, i.e., for $\alpha_0 = 10$, 20, and 30 deg. However, it has been observed that for larger angle of incidence the curves attain maximum at higher shock strengths. The trend of the curves for m_1 is similar to the normal shock case as demonstrated by Skews.[3]

In Fig. 4, m_2 is plotted against ξ. The curves show monotonic increase from $\xi = 0$ to $\xi = 1$ for all three angles of incidence ($\alpha_0 =$ 10, 20, and 30 deg). For fixed incident shock strengths, the higher is the angle of incidence, the lower is the value of m_2.

The experimental results of Srivastava and Deschambault[5] for $\xi = 0.5$ and $\alpha_0 = 20$ deg tally fairly well with the values of m_1 and m_2 predicted by theory. Experimental results appear to be smaller than the theoretical results. The analytical shock Mach number corresponding to $\xi = 0.5$ is 1.36, whereas the experimental shock Mach number was 1.39. The results are in good agreement despite this small difference.

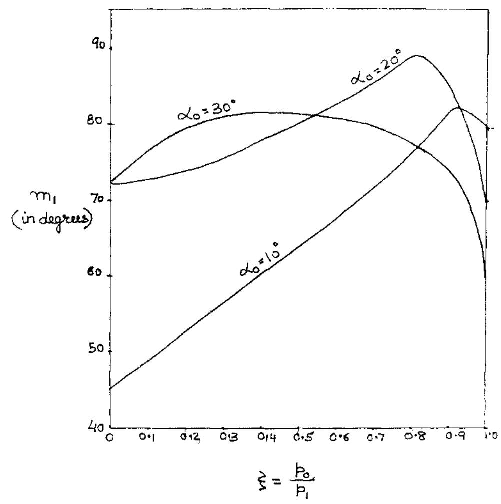

Fig. 3 Angle m_1 for the start of curvature vs shock strength.

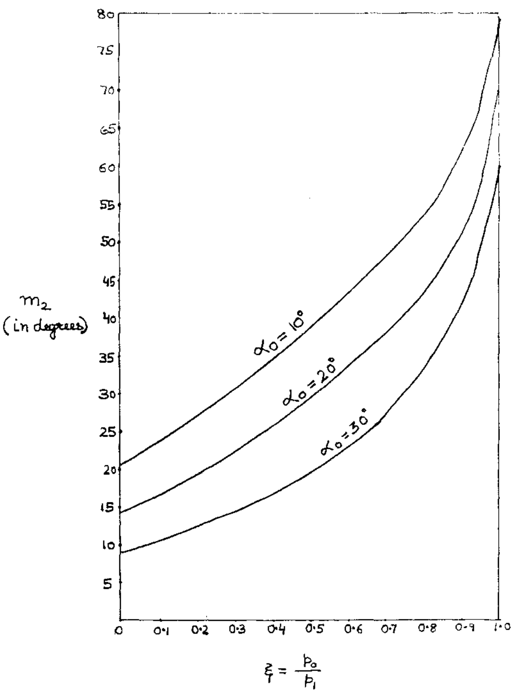

Fig. 4 Angle m_2 for the start of curvature vs shock strength.

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